

Stat 20 Discussion for section #3

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Question 1

We play a Wheel of Fortune game. It costs \$1 to play the game. There are 6 slots of unequal probability:

Slot	Probability	Pay off	Net
1	0.6	0	-1
2	0.2	1	0
3	0.12	2	1
4	0.05	3	2
5	0.02	10	9
6	0.01	20	19

Let W_1 be amount of net win (pay off -\$1).

Let W_2 be amount of net win or 2nd.

Let $W = W_1 + W_2$

What is

- (a) $P(W_1 = W_2)$?
- (b) $P(W_1 > W_2)$?
- (c) $E(W_1)$ $\text{Var}(W_1)$?
- (d) $E(W_1)$ $\text{Var}(W)$?

Answer:

$$\begin{aligned} \text{(a) } P(W_1 = W_2) &= P(W_1 = W_2 = -1) + \dots + P(W_1 = W_2 = 19) \\ &= 0.6^2 + 0.2^2 + 0.12^2 + 0.05^2 + 0.02^2 + 0.01^2 \\ &= 0.36 + 0.04 + 0.144 + 0.0025 + 0.0004 + 0.0001 \\ &= 0.547 \end{aligned}$$

$$\begin{aligned} \text{(b) } P(W_1 > W_2) &= P(W_2 > W_1) = \frac{1 - P(W_1 = W_2)}{2} \\ &= \frac{1 - 0.547}{2} = \frac{0.453}{2} \\ &= 0.2265 \end{aligned}$$

$$\begin{aligned}
 \text{(c) } E(W_1) &= -1(0.6) + 0(0.2) + 1(0.12) + 2(0.05) + 9(0.02) + 19(0.01) \\
 &= -0.06 + 0.12 + 0.1 + 0.18 + 0.19 \\
 &= -0.01
 \end{aligned}$$

$$\text{Var}(W_1) = E(W_1)^2 - (E(W_1))^2$$

$$\begin{aligned}
 E(W_1)^2 &= (-1)^2(0.6) + 0^2(0.2) + 1(0.12) + 4(0.05) + 81(0.02) + 361(0.01) \\
 &= 0.6 + 0 + 0.12 + 0.2 + 1.62 + 3.61 \\
 &= 6.15
 \end{aligned}$$

$$\text{Var}(W_1) = 6.15 - (-0.01)^2 = 6.1499$$

$$\text{(d) } E(W) = E(W_1) + E(W_2) = -0.01 - 0.01 = -0.02$$

$$\text{Var}(W) = \text{Var}(W_1 W_2) = \text{Var}(W_1) + \text{Var}(W_2) = 6.1499 + 6.1499 = 12.2998$$

Question 2

Let $Z_1, Z_2,$ and Z_3 be independent random variables, each with mean 0, variance 1.

$$\text{Let } X = Z + 3Z_1 + 4Z_2 + Z_3$$

$$Y = 9 - 3Z_1 + Z_2 + 4Z_3$$

- (a) What is the mean and variance of X?
- (b) What is the mean and variance of Y?
- (c) What is the mean and variance of $2X + Y$?

Answer: Discussed in class. Note that for (c) X and Y are not independent.

Question 3

X, Y, and Z are independent random variables with SD O. Let a, b, and c be constants such that $a^2 + b^2 + c^2 > 0$.

What is the variance of $\frac{ax - by - cz}{a^2 + b^2 + c^2}$?

Answer: Discussed in class.

Question 4

We have two scales for measuring weights in a chemistry lab. Repeated weighings of an item on the same scale produces varying measurements.

Scale 1: mean 2.000g SD 0.002

Scale 2: mean 2.001g SD 0.001

Let X be the measurement on scale 1. Let Y be the measurement on scale 2. The readings are independent between scales. What is $E(Y - X)$? $\text{Var}(Y - X)$?

Let $Z = (X + Y)/2$. What is $E(Z)$, $\text{Var}(Z)$?

Is the average (Z) more or less variable than Y (the less variable value)?

Answer:

$$E(Y - X) = 2.001 - 2.000 = 0.001$$

$$\begin{aligned}\text{Var}(Y - X) &= \text{Var}(Y) + \text{Var}(X) \quad (\text{independent}) \\ &= (0.002)^2 + (0.001)^2 \\ &= 0.000005\end{aligned}$$

$$E(Z) = E((X + Y)/2) = \frac{1}{2} (E(X) + E(Y)) = 2.0005$$

$$\text{Var}(Z) = \text{Var}(1/2(X + Y)) = \frac{1}{4} (\text{Var}(X) + \text{Var}(Y)) = \frac{1}{4} (0.000005) = 0.00000125$$

The average Z is less variable than either variable X or Y .

Question 5

Consider two 1 year investments (each of \$1000). For the 1st investment we get back \$1000 with $p = 0.6$, \$2000 with $p = 0.3$, and \$0 with $p = 0.1$. For the 2nd investment, \$1100 with $p = 0.6$, \$1300 with $p = 0.3$, and \$500 with $p = 0.1$.

- (a) For which investment do we expect to get back more?
- (b) Which investment gives us the most variable return?

Answer:

- (a) For the 1st investment. The expected value is $0.1 * (0) + 0.6 * 1000 + 0.3 * (2000) = \1200 and for the 2nd investment. The expected value is $0.1 * (500) + 0.6 * (1100) + 0.3 * (1300) = \1100 . So the first investment will give us back more (on average).
- (b) For the 1st investment the variance is $0.1 * (0)^2 + (0.6) * (1000)^2 + 0.3 * (2000)^2 = 1,800,000$. For the 2nd investment the variance is $0.1 * (500)^2 + (0.6) * (1100)^2 + (0.3) * (1300)^2 = 1,258,000$. So the first investment is more variable.