

Stat 20: Discussion at section #4

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Question 1

A factory employs several thousand workers of whom 30% are Hispanic. If the 15 members of the union executive committee are chosen from all the workers at random.

- (a) What is the probability that exactly 3 members are Hispanic?
- (b) What is the probability of less than 3 Hispanics being members of the committee?
- (c) What is the probability of 3 or more members of the committee being Hispanic?
- (d) What is the mean and variance of the number of Hispanics selected for the committee?

Answer: Let Y = number of Hispanics on committee. Then Y is binomial with $n = 15$ and $p = 0.3$

(a) $P(Y = 3) = \binom{15}{3} 0.3^3 (1-0.3)^{15-3} = \frac{15!}{12!3!} 0.3^3 (0.7)^{12} = \frac{15(14)(13)}{3(2)(1)} 0.3^3 (0.7)^{12} = 0.170$ (3dp)

(b)

$$\begin{aligned} P(Y < 3) &= P(Y = 0) + P(Y = 1) + P(Y = 2) \\ &= \binom{15}{0} 0.3^0 (1 - 0.3)^{15} + \binom{15}{1} 0.3^1 (1 - 0.3)^{15-1} + \binom{15}{2} 0.3^2 (1 - 0.3)^{15-2} \\ &= (0.7)^{15} + 13(0.3)(0.7)^{14} + \frac{15(14)}{2} 0.3^2 (0.7)^{13} \\ &= 0.127 \text{ (3dp)} \end{aligned}$$

(c) $P(Y \geq 3) = 1 - P(Y < 3) = 1 - 0.127 = 0.873$ (3dp)

(d) $E(Y) = np = 15(.3) = 4.5$ and $Var(Y) = np(1 - p) = 15(0.3)(0.7) = 3.15$

Question 2

Suppose we toss a fair coin 10 times and roll a fair six sided dice 6 times then what is the probability of getting 7 heads and the dice showing a 1 or 6 on three of the rolls?

Answer: Let X = number of heads in 10 tosses. So X is distributed Binomial $n_1 = 10$, $p_1 = 0.5$. Let Y = number of times 1 or 6 shows in 6 rolls. So Y is distributed Binomial $n_1 = 6$, $p_1 = 1/3$.

Then $P(X = 7) = \binom{10}{7} 0.5^7(1 - 0.5)^3 = 0.117(3 \text{ dp})$ and $P(Y = 3) = \binom{6}{3} (1/3)^3(1 - 1/3)^3 = 0.219(3 \text{ dp})$.

Since X and Y are independent. $P(X = 7 \cap Y = 3) = P(X = 7)P(Y = 3) = (0.117)(0.219) = 0.026(3 \text{ dp})$.

Question 3

Suppose we toss a fair penny 4 times and a bent quarter 6 times ($P(H) = 0.75$). What is the expected number of heads? What is the variance?

Answer: Let

X = Number of heads on penny. X is binomial $n_1 = 4$, $P_1 = 0.5$

Y = Number of heads on quarter. Y is binomial $n_1 = 6$, $P_1 = 0.75$

$T = X + Y$ be the total number of heads.

Then $E(X) = 4(.5) = 2$ and $E(Y) = 6(.75) = 4.5$. Similarly $Var(X) = 4(.5)(.5) = 1$ and $Var(Y) = 6(.75)(.25) = 1.125$.

Since $T = X + Y$. It follows that $E(T) = E(X + Y) = E(X) + E(Y) = 2 + 4.5 = 6.5$. The results from tossing each of the coins are independent therefore $Var(T) = Var(X + Y) = Var(X) + Var(Y) = 1 + 1.125 = 2.125$.

Question 4

Returning to our coin tossing game of previous sections. Remember that this is a game of 4 tosses where we win \$1 for each heads but lose \$1 for each tails.

- What is the mean number of heads? How about the variance?
- What is our expected winnings? How about the variance?
- How about with an unfair coin ($P(H) = 0.25$). What are (a) and (b) in this case?
- How about with an unfair coin and we play for 20 tosses?

Answer: Let X = number of heads in game. So X is Binomial with $p = 0.5$, $n = 4$.

- $E(X) = 4(.5) = 2$ and $Var(X) = 4(.5)(.5) = 1$.

Let W be winnings in \$ from playing games. Since we win \$1 from each H and lose \$1 from each tail (of which there will be $n - X$) our total winnings is $W = (1)X + (-1)(n - X) = 2X - n$.

- (b) $E(W) = E(2X - n) = 2E(X) - n = 2(2) - 4 = 0$ and $Var(W) = Var(2X - n) = 4Var(X) = 4$.
- (c) Now $p = 0.25$ and so $E(X) = 4(.25) = 1$ and $Var(X) = 4(.25)(.75) = 3/4$. So, $E(W) = E(2X - n) = 2E(X) - n = 2(1) - 4 = -2$ and $Var(W) = Var(2X - n) = 4Var(X) = 3$.
- (d) Now $p = 0.25$ and $n = 20$ Thus, $E(X) = 20(.25) = 5$ and $Var(X) = 20(.25)(.75) = 3.75$. So, $E(W) = E(2X - n) = 2E(X) - n = 2(5) - 20 = -10$ and $Var(W) = Var(2X - n) = 4Var(X) = 15$.

Miscellaneous

Remember the following for the Binomial distribution.

1. If Y is distributed Binomial with parameters n, p write $Y \sim \text{Bin}(n, p)$.
2. Probability distribution function: $P(Y = k) = \binom{n}{p} p^k (1 - p)^{n-k}$
3. Expected Value: $E(Y) = np$
4. Variance: $Var(Y) = np(1 - p)$

Note that $\binom{n}{p} = \frac{n!}{k!(n-k)!}$. Where $n! = n(n-1)(n-2) \dots 1$. Also remember that $0! = 1$.